# Uniqueness theorem for generalized Maxwell electric and magnetic black holes in higher dimensions

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Based on the conformal energy theorem we prove the uniqueness theorem for static higher dimensional *electrically* and *magnetically* charged black holes being the solution of Einstein $(n-2)$-gauge forms equations of motion. Black holes spacetime contains an asymptotically flat spacelike hypersurface with compact interior and nondegenerate components of the event horizon.

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## I. INTRODUCTION

The problem of classification of nonsingular black hole solutions was first raised by Israel [1], Müller zum Hagen *et al.* [2], and Robinson [3], while the most complete results were proposed in Refs. [4–8]. The classification of static vacuum black hole solutions was finished in [9], where the condition of nondegeneracy of the event horizon was removed. In Ref. [10] the Einstein-Maxwell (EM) black holes were treated and it was shown that for the static electrovacuum black holes all degenerate components of the event horizon should have charges of the same signs.

The problem of the uniqueness black hole theorem for stationary axisymmetric spacetime was considered in Refs. [11]. But the complete proof was delivered by Mazur [12] and Bunting [13] (see for a review of the uniqueness of black hole solutions story see [14] and references therein).

Due to the attempts of building a consistent quantum gravity theory there was also resurgence of works concerning the mathematical aspects of the low-energy string theory black holes. The staticity theorem for Einstein-Maxweel axion dilaton (EMAD) gravity was studied in Ref. [15]. Then, the uniqueness of the black hole solutions in dilaton gravity was proved in works [16,17], while the uniqueness of the static dilaton $U(1)^2$ black holes being the solution of $N=4,d=4$ supergravity was provided in [18]. The extension of the proof to the theory to allow for the inclusion of $U(1)^N$ static dilaton black holes was established in Ref. [19].

The recent unification attempts such as M or string theory reveal the concept that our Universe may be a brane or defect emerged in higher dimensional geometry. $E8\times E8$ heterotic string theory at strong coupling may be described in terms of M-theory acting in eleven-dimensional spacetime with boundaries where ten-dimensional Yang-Mills gauge theories reside on two boundaries [20]. One hopes, that this idea will be helpful in solving hierarchy problem. All these trigger the interests in higher dimensional black hole solutions. The so-called TeV gravity attracts attention to higher dimensional black hole which may be produced in high energy experiments [21].

The considerable interest was also attributed to $n$-dimensional black hole uniqueness theorem, both in vacuum and charged case [24–27]. The complete classification of $n$-dimensional charged black holes having both degenerate and nondegenerate components of event horizon was provided in Ref. [28]. Proving the uniqueness theorem for stationary $n$-dimensional black holes is much more complicated. It turned out that generalization of Kerr metric to arbitrary $n$-dimensions proposed by Myers-Perry [22] is not unique. The counterexample showing that a five-dimensional rotating black hole ring solution with the same angular momentum and mass but the horizon of which was homeomorphic to $S^2\times S^1$ was presented in [23] (see also Ref. [29]). Recently, it has been established that Myers-Perry solution is the unique black hole in five-dimensions in the class of spherical topology and three commuting Killing vectors [30].

The uniqueness theorem for self-gravitating nonlinear $\sigma$ models in higher dimensional spacetime was obtained in [31]. The $n$-dimensional black hole uniqueness theorems for supersymmetric black holes in five-dimensions were given in Refs. [32].

In Ref. [22] it was pointed out that a black hole being the source of both magnetic and electric components of 2-form $F_{\mu\nu}$ was a striking coincidence. In order to treat this problem in $n$-dimensional gravity we shall consider both *electric* and *magnetic* components of $(n-2)$-gauge form $F_{\mu_1 \dots \mu_{n-2}}$. We shall comprise the uniqueness of *electrically* and *magnetically* charged static $n$-dimensional black hole solution containing an asymptotically flat hypersurface with compact interior with nondegenerate components of the event horizon. The main result of our paper will be the proof of the uniqueness of static higher dimensional *electrically* and *magnetically* charged black hole containing an asymptotically flat hypersurface with compact interior and nondegenerate components of the event horizon.

## II. HIGHER DIMENSIONAL GENERALIZED EINSTEIN-MAXWELL SYSTEM

In this section we shall examine the generalized Maxwell $(n-2)$-gauge form $F_{\mu_1 \dots \mu_{n-2}}$ in $n$-dimensional spacetime described by the following action:

$$I=\int d^n x \sqrt{-g}[\,{}^{(n)}R - F^2_{(n-2)}], \qquad (1)$$

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where $g_{\mu\nu}$ is $n$-dimensional metric tensor, $F_{(n-2)}=dA_{(n-3)}$ is $(n-2)$-gauge form field. The metric of $n$-dimensional static spacetime subject to the asymptotically timelike Killing vector field $k_\alpha=(\partial/\partial t)_\alpha$ and $V^2=-k_\mu k^\mu$ can be written in the following form:

$$ds^2=-V^2dt^2+g_{ij}dx^idx^j, \tag{2}$$

where $V$ and $g_{ij}$ are independent of the $t$-coordinate as the quantities of the hypersurface $\Sigma$ of constant $t$.

The energy momentum tensor of the $(n-2)$-gauge form $T_{\mu\nu}=-\delta S/\sqrt{-g}\,\delta g^{\mu\nu}$ yields

$$T_{\mu\nu}=(n-2)F_{\mu i_2 \dots i_{n-2}}F_\nu{}^{i_2 \dots i_{n-2}}-\frac{g_{\mu\nu}}{2}F^2_{(n-2)}. \tag{3}$$

In our consideration we shall take into account the asymptotically flat spacetime, i.e., the spacetime will contain a data set $(\Sigma_{end},g_{ij},K_{ij})$ with gauge fields of $F_{(n-2)}$ such that $\Sigma_{end}$ is diffeomorphic to $\mathbf{R}^{n-1}$ minus a ball. The asymptotical conditions of the following forms should also be satisfied:

$$|g_{ij}-\delta_{ij}|+r|\partial_a g_{ij}|+\dots+r^m|\partial_{a_1 \dots a_m}g_{ij}|+r|K_{ij}|+\dots + r^m|\partial_{a_1 \dots a_m}K_{ij}|\leqslant\mathcal{O}\left(\frac{1}{r}\right), \tag{4}$$

$$|F_{i_1 \dots i_{n-2}}|+r|\partial_a F_{i_1 \dots i_{n-2}}|+\dots+r^m|\partial_{a_1 \dots a_m}F_{i_1 \dots i_{n-2}}| \leqslant\mathcal{O}\left(\frac{1}{r^2}\right). \tag{5}$$

We define *electric* $(n-3)$-form by the following expression:

$$E_{i_1 \dots i_{n-3}}=F_{i_1 \dots i_{n-2}}k^{i_{n-2}}, \tag{6}$$

and *magnetic* 1-form as

$$B_k=\frac{1}{\sqrt{2(n-2)!}}\epsilon_{k\mu i_1 \dots i_{n-2}}F^{i_1 \dots i_{n-2}}k^\mu. \tag{7}$$

We introduce also the rotation $(n-3)$-form of the stationary Killing vector field $k_\mu$

$$\omega_{j_2 \dots j_{n-2}}=\frac{(n-2)}{\sqrt{2(n-2)!}}\epsilon_{j_2 \dots j_{n-2}\mu\nu\gamma}k^\mu\nabla^\nu k^\gamma. \tag{8}$$

Directly from the definition (8) and definition of *electric* and *magnetic* forms and equations of motion for $(n-2)$-gauge form we find equations of motion for *magnetic* 1-form $B_k$

$$\nabla^a\left(\frac{B_a}{N}\right)=\frac{E^{i_2 \dots i_{n-2}}\omega_{i_2 \dots i_{n-2}}}{N^2}, \tag{9}$$

and similarly for *electric* $(n-3)$-form $E_{i_1 \dots i_{n-3}}$

$$\nabla^{j_1}\left(\frac{E_{j_1 j_2 \dots j_{n-3}}}{N}\right)=-\frac{B^a\omega_{a j_2 \dots j_{n-2}}}{N^2}, \tag{10}$$

where we have denoted $N=k_\mu k^\mu$.

In an asymptotically flat, globally hyperbolic spacetime with compact bifurcation surface and strictly stationary, simply connected domain of outer communication $\langle\langle\mathcal{J}\rangle\rangle$ (i.e., $V^2=-k_\mu k^\mu\geqslant 0$) one has that $\langle\langle\mathcal{J}\rangle\rangle$ is static if

$$\omega_{j_2 \dots j_{n-2}}=0 \quad\Leftrightarrow\quad k^\alpha R_{\alpha[\beta}k_{\gamma]}=0. \tag{11}$$

This statement can be verified by the direct use of equations of motion and by using the stationarity conditions for $F_{\mu_1\mu_2 \dots \mu_{n-2}}$ with respect to the Killing vector field $k_\mu$, namely

$$\mathcal{L}_{k^\mu}F_{\mu_1 \dots \mu_{n-2}}=0. \tag{12}$$

Then, by the direct calculation one can check that $R_{\alpha\beta}k^\beta=0$ and this provides the statement. In our paper we deal with the static charged black hole case which implies that in static domain of outer communication $\langle\langle\mathcal{J}\rangle\rangle$ the right-hand sides of Eqs. (9) and (10) are equal to zero. Having in mind the exact form of the metric tensor (2) on the hypersurface $\Sigma$ orthogonal to the Killing vector field $k_\mu$ and assuming the only one nontrivial *electric* component of the $(n-2)$-form $F_{\mu_1 \dots \mu_{n-2}}$ as $A_{01 \dots n-2}=\phi(x)$ and for the *magnetic* potential $B_k={}^{(g)}\nabla_k g(x)$, we get the following equations of motion:

$$^{(g)}\nabla_i{}^{(g)}\nabla^i V=\frac{(n-3)^2}{V}{}^{(g)}\nabla_i\phi\,{}^{(g)}\nabla^i\phi + \frac{2(n-3)}{(n-2)V}{}^{(g)}\nabla_i g\,{}^{(g)}\nabla^i g, \tag{13}$$

$$^{(g)}\nabla_i{}^{(g)}\nabla^i \phi=\frac{1}{V}{}^{(g)}\nabla_i\phi\,{}^{(g)}\nabla^i V, \tag{14}$$

$$^{(g)}\nabla_i{}^{(g)}\nabla^i g=\frac{1}{V}{}^{(g)}\nabla_i g\,{}^{(g)}\nabla^i V, \tag{15}$$

$$^{(n-1)}R_{ij}=\frac{^{(g)}\nabla_i{}^{(g)}\nabla_j V}{V}-\frac{n-2}{V^2}{}^{(g)}\nabla_i\phi\,{}^{(g)}\nabla_j\phi + \frac{g_{ij}}{V^2}{}^{(g)}\nabla_i\phi\,{}^{(g)}\nabla^i\phi-\frac{2}{V^2}{}^{(g)}\nabla_i g\,{}^{(g)}\nabla_j g + \frac{2g_{ij}}{(n-2)V^2}{}^{(g)}\nabla_i g\,{}^{(g)}\nabla^i g. \tag{16}$$

The covariant derivative with respect to $g_{ij}$ is denoted by ${}^{(g)}\nabla$, while ${}^{(n-1)}R_{ij}(g)$ is the Ricci tensor defined on the hypersurface $\Sigma$.

Let us assume further that in asymptotically flat spacetime there is a standard coordinates system in which we have the usual asymptotic expansion

$$V=1-\frac{\mu}{r^{n-3}}+\mathcal{O}\left(\frac{1}{r^{n-2}}\right), \tag{17}$$

and for the metric tensor

$$g_{ij}=\left(1+\frac{2}{n-3}\frac{\mu}{r^{n-3}}\right)+\mathcal{O}\left(\frac{1}{r^{n-2}}\right). \quad (18)$$

While for *electric* and *magnetic* potential one gets respectively

$$\phi=\frac{Q/C_1}{r^{n-3}}+\mathcal{O}\left(\frac{1}{r^{n-2}}\right), \quad (19)$$

$$g=\frac{P/C_2}{r^{n-3}}+\mathcal{O}\left(\frac{1}{r^{n-2}}\right), \quad (20)$$

where $C_1=n-3$, $C_2=2(n-3)/n-2$, $\mu$ is the ADM mass seen by the observer from the infinity, $Q$ is the electric charge, $P$ is magnetic charge while $r^2=x_i x^i$.

Let us define the quantities in the forms as follows:

$$\Phi_1=\frac{1}{2}\left[V+\frac{1}{V}-\frac{(n-2)\phi^2}{V}\right], \quad (21)$$

$$\Phi_0=\frac{\sqrt{n-2}\,\phi}{V}, \quad (22)$$

$$\Phi_{-1}=\frac{1}{2}\left[V-\frac{1}{V}-\frac{(n-2)\phi^2}{V}\right], \quad (23)$$

and

$$\Psi_1=\frac{1}{2}\left[V+\frac{1}{V}-\frac{2(n-3)g^2}{V}\right], \quad (24)$$

$$\Psi_0=\sqrt{\frac{2(n-3)g}{V}}, \quad (25)$$

$$\Psi_{-1}=\frac{1}{2}\left[V-\frac{1}{V}-\frac{2(n-3)g^2}{V}\right]. \quad (26)$$

Furthermore, if we define the metric $\eta_{AB}=\mathrm{diag}(1,-1,-1)$, one can check that

$$\Phi_A\Phi^A=\Psi_A\Psi^A=-1. \quad (27)$$

Next consider the following conformal transformation:

$$\tilde{g}_{ij}=V^{\frac{2}{n-3}}g_{ij}, \quad (28)$$

and introduce the symmetric tensors written as

$$\tilde{G}_{ij}=\tilde{\nabla}_i\Phi_{-1}\tilde{\nabla}_j\Phi_{-1}-\tilde{\nabla}_i\Phi_0\tilde{\nabla}_j\Phi_0-\tilde{\nabla}_i\Phi_1\tilde{\nabla}_j\Phi_1, \quad (29)$$

and similarly for the potential $\Psi_A$

$$\tilde{H}_{ij}=\tilde{\nabla}_i\Psi_{-1}\tilde{\nabla}_j\Psi_{-1}-\tilde{\nabla}_i\Psi_0\tilde{\nabla}_j\Psi_0-\tilde{\nabla}_i\Psi_1\tilde{\nabla}_j\Psi_1, \quad (30)$$

where $\tilde{\nabla}_i$ is the covariant derivative with respect to the metric $\tilde{g}_{ij}$. By virtue of relations (29) and (30) the field equations imply

$$\tilde{\nabla}^2\Phi_A=\tilde{G}_i{}^i\Phi_A, \quad \tilde{\nabla}^2\Psi_A=\tilde{H}_i{}^i\Psi_A, \quad (31)$$

where $A=-1,0,1$, and it is straightforward to establish that the Ricci tensor of the metric $\tilde{g}_{ij}$ can be expressed as

$$\tilde{R}_{ij}=\tilde{G}_{ij}+\frac{1}{n-3}\tilde{H}_{ij}. \quad (32)$$

In the next step we study the conformal transformations given by the expressions

$${}^{\Phi}g^{\pm}_{ij}={}^{\phi}\omega_{\pm}^{2/(n-3)}\tilde{g}_{ij}, \quad {}^{\Psi}g^{\pm}_{ij}={}^{\psi}\omega_{\pm}^{2/(n-3)}\tilde{g}_{ij}, \quad (33)$$

where the conformal factors are determined by

$${}^{\Phi}\omega_{\pm}=\frac{\Phi_1\pm 1}{2}, \quad {}^{\Psi}\omega_{\pm}=\frac{\Psi_1\pm 1}{2}. \quad (34)$$

Just one gets four manifolds $(\Sigma_+^{\Phi},{}^{\Phi}g_{ij}^{+})$, $(\Sigma_-^{\Phi},{}^{\Phi}g_{ij}^{-})$, $(\Sigma_+^{\Psi},{}^{\Psi}g_{ij}^{+})$, $(\Sigma_-^{\Psi},{}^{\Psi}g_{ij}^{+})$. Pasting $(\Sigma_{\pm}^{\Phi},{}^{\Phi}g_{ij}^{\pm})$ and $(\Sigma_{\pm}^{\Psi},{}^{\Psi}g_{ij}^{\pm})$ across the surface $V=0$ we can construct regular hypersurfaces $\Sigma^{\Phi}=\Sigma_+^{\Phi}\cup\Sigma_-^{\Phi}$ and $\Sigma^{\Psi}=\Sigma_+^{\Psi}\cup\Sigma_-^{\Psi}$. If $(\Sigma,g_{ij},\Phi_A,\Psi_A)$ are asymptotically flat solution of Eqs. (31) and (32) with nondegenerate black hole event horizon, our next task will be to check that total gravitational mass on hypersurfaces $\Sigma^{\Phi}$ and $\Sigma^{\Psi}$ is equal to zero. In order to do this we shall implement the conformal positive mass theorem in higher dimensions [25,33]. Now using another conformal transformation given by

$$\hat{g}^{\pm}_{ij}=[({}^{\Phi}\omega_{\pm})^2({}^{\Psi}\omega_{\pm})^{2\lambda}]^{1/(n-3)(1+\lambda)}\tilde{g}_{ij}, \quad (35)$$

it follows that the Ricci curvature tensor on the space yields

$$(1+\lambda)\hat{R}=[{}^{\Phi}\omega_{\pm}^2\,{}^{\Psi}\omega_{\pm}^{2\lambda}]^{-1/(n-3)(1+\lambda)}({}^{\Phi}\omega_{\pm}^{[2/(n-3)]}\,{}^{\Phi}R$$
$$+\lambda\,{}^{\Psi}\omega_{\pm}^{[2/(n-3)]}\,{}^{\Psi}R)+\frac{\lambda}{1+\lambda}\left(\frac{n-2}{n-3}\right)(\hat{\nabla}_i\ln{}^{\Phi}\omega_{\pm}$$
$$-\hat{\nabla}_i\ln{}^{\Psi}\omega_{\pm})(\hat{\nabla}^i\ln{}^{\Phi}\omega_{\pm}-\hat{\nabla}^i\ln{}^{\Psi}\omega_{\pm}). \quad (36)$$

For this stage on we shall take $\lambda=1/n-3$.

The close inspection of the first term in Eq. (36) reveals that it is non-negative. Namely, one can establish that it may be written in the form as follows:

$$
{}^{\Phi}\omega_{\pm}^{[2/(n-3)]}\, {}^{\Phi}R + \lambda\, {}^{\Psi}\omega_{\pm}^{[2/(n-3)]}\, {}^{\Psi}R = \left(\frac{n-2}{n-3}\right)\left|\frac{\Phi_0 \tilde{\nabla}_i \Phi_{-1} - \Phi_{-1}\tilde{\nabla}_i \Phi_0}{\Phi_1 \pm 1}\right|^2 + \frac{(n-2)}{(n-3)^2}\left|\frac{\Psi_0 \tilde{\nabla}_i \Psi_{-1} - \Psi_{-1}\tilde{\nabla}_i \Psi_0}{\Psi_1 \pm 1}\right|^2. \qquad (37)
$$

Applying the conformal energy theorem we draw a conclusion that $(\Sigma^{\Phi}, {}^{\Phi}g_{ij})$, $(\Sigma^{\Psi}, {}^{\Psi}g_{ij})$ and $(\hat{\Sigma}, \hat{g}_{ij})$ are flat and it in turns implies that the conformal factors ${}^{\Phi}\omega = {}^{\Psi}\omega$ and $\Phi_1 = \Psi_1$. Furthermore $\Phi_0 = \text{const}\, \Phi_{-1}$ and $\Psi_0 = \text{const}\, \Psi_{-1}$. Just the above potentials are functions of a single variable. Moreover, the manifold $(\Sigma, g_{ij})$ is conformally flat. We can rewrite $\hat{g}_{ij}$ in a conformally flat form [24,25], i.e., we define a function

$$
\hat{g}_{ij} = \mathcal{U}^{[4/(n-3)]}\, {}^{\Phi}g_{ij}, \qquad (38)
$$

where $\mathcal{U} = ({}^{\Phi}\omega_{\pm} V)^{-1/2}$. Just, it turned out that $\hat{R}$ is equal to zero provided the Einstein $(n-2)$-gauge form equations of motion reduced to the Laplace equation on the $(n-1)$ Euclidean manifold

$$
\nabla_i \nabla^i \mathcal{U} = 0, \qquad (39)
$$

where $\nabla$ is the connection on a flat manifold. Having in mind the above equation we can imply the following expression for the flat base space:

$$
{}^{\Phi}g_{ij} dx^i dx^j = \tilde{\rho}^2 d\mathcal{U}^2 + \tilde{h}_{AB} dx^A dx^B. \qquad (40)
$$

The event horizon is located at some $\mathcal{U} = \text{const}$ and one can show that the embedding of $\mathcal{H}$ into the Euclidean $(n-1)$ space is totally umbilical [34]. Each of the connected components of the horizon $\mathcal{H}$ will be a geometric sphere of a certain radius. The radius can be determined by the value of $\rho|_{\mathcal{H}}$, where $\rho$ is the coordinate which can be introduced on $\Sigma$ as follows:

$$
\hat{g}_{ij} dx^i dx^j = \rho^2 dV^2 + h_{AB} dx^A dx^B.
$$

Thus, it is clearly seen that the embedding is hyperspherical.

Of course one can always locate one connected component of the horizon at $r = r_0$ surface without loss of generality. Thus we have a boundary value problem for the Laplace equation on the base space $\Omega = E^{n-1}/B^{n-1}$ with the Dirichlet boundary condition (the rigid embedding [34]). Let $\mathcal{U}_1$ and $\mathcal{U}_2$ be two solutions of the boundary value problem. By successive use of the Green identity and integrating over the volume element we find

$$
\left(\int_{r\to\infty} - \int_{\mathcal{H}}\right)(\mathcal{U}_1 - \mathcal{U}_2)\frac{\partial}{\partial r}(\mathcal{U}_1 - \mathcal{U}_2) dS = \int_{\Omega} |\nabla(\mathcal{U}_1 - \mathcal{U}_2)|^2 d\Omega. \qquad (41)
$$

The surface integrals vanish due to the imposed boundary conditions provided that the volume integral must be identically equal to zero.

Hence the preceding results can be collected in the following:

*Theorem*: Consider a static solution to $n$-dimensional Einstein $(n-2)$-gauge forms $F_{\mu_1 \dots \mu_{n-2}}$ equation of motion with only *electric* and *magnetic* charge. Let us suppose that we have an asymptotically timelike Killing vector field $k_\mu$ orthogonal to the connected and simply connected spacelike hypersurface $\Sigma$. The topological boundary $\partial\Sigma$ of $\Sigma$ is a nonempty topological manifold with $g_{ij}k^i k^j = 0$ on $\partial\Sigma$. Thus, we obtain the following conclusion.

If $\partial\Sigma$ is connected, then there exist a neighborhood of the hypersurface $\Sigma$ which is diffeomorphic to an open set of a generalized Reissner-Nordsröm nonextreme solution with *electric* and *magnetic* charges provided by the adequate *electric* and *magnetic* components of the gauge $(n-2)$-form $F_{\mu_1 \dots \mu_{n-2}}$.


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